

Can be homogeneous

From

$$(a_1x + b_1y + c_1)dx + (a_2x + b_2y + c_2)dy = 0$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

نعم، فإن المعادلات المتجانسة (homogeneous) تكون بالشكل  $(a_1x + b_1y + c_1)dx + (a_2x + b_2y + c_2)dy = 0$

$$X = x + h$$

$$Y = y + k$$

$$\begin{aligned} a_1h + b_1k + c_1 &= 0 \\ a_2h + b_2k + c_2 &= 0 \end{aligned} \Rightarrow (a_1x + b_1y)dx + (a_2x + b_2y)dy = 0$$

ex: 1

$$\frac{dy}{dx} = \frac{4x + 2y - 10}{2x - y - 3}$$

(h, k) نضعه في المعادلات

$$\therefore 4h + 2k - 10 = 0 \rightarrow (1)$$

$$2h - k - 3 = 0 \rightarrow (2)$$

$$\therefore k = 1 \quad h = 2$$

$$X = x + 2$$

$$Y = y + 1$$

$$\therefore \frac{dY}{dX} = \frac{4X + 2Y}{2X - Y}$$

$$x = vx \quad \therefore y' = v + x \frac{dv}{dx}$$

$$\therefore v + x \frac{dv}{dx} = \frac{4x + 2vx}{2x - vx}$$

$$\therefore v + x \frac{dv}{dx} = \frac{x(4+2v)}{x(2-v)}$$

$$\therefore x \frac{dv}{dx} = \frac{4+2v}{2-v} - v$$

$$\therefore x \frac{dv}{dx} = \frac{4+2v-2v+v^2}{2-v} = \frac{4+v^2}{2-v}$$

$$\therefore x \frac{dv}{dx} = \frac{4+v^2}{2-v} \quad \therefore \int \frac{1}{x} dx = \int \frac{2-v}{4+v^2} dv$$

$$\therefore \ln x + C = \int \left( \frac{2}{4-v^2} \right) dv - \int \frac{1}{2} \left( \frac{v}{4+v^2} \right) dv$$

$$\ln x + C = 2 \tan^{-1} \left( \frac{v}{2} \right) - \frac{1}{2} \ln(4-v^2)$$

$$\therefore 2 \tan^{-1} \left( \frac{y}{2x} \right) - \frac{1}{2} \ln \left( 4 + \left( \frac{y}{x} \right)^2 \right) = \ln x + C$$

$$x = x-2 \quad y' = y-1$$

$$\therefore 2 \tan^{-1} \frac{y-1}{2(x-2)} - \frac{1}{2} \ln \left( 4 + \left( \frac{y-1}{x-2} \right)^2 \right) = \ln(x-2) + C$$



ExactForm  $M(x,y)dy + N(x,y)dX = 0$ 

$$\Rightarrow y' = -\frac{N}{M}$$

iff  $\Rightarrow$ 

$$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$$

شروط exact

\* نتأكد من ان الفرق بين الطرفين يساوي صفر

ex:  $(y \cos xy + e^x) dy + (x \cos xy - 2y e^{y^2}) dx = 0$

Solution

$$M = y \cos xy + e^x \quad \therefore \frac{\partial M}{\partial x} = -y^2 \sin xy + e^x$$

$$N = x \cos xy - 2y e^{y^2} \quad \therefore \frac{\partial N}{\partial y} = -x^2 \sin xy - 2y + 2y e^{y^2} + -2e^{y^2}$$

ex  $(y \cos xy + e^x) dx + (x \cos xy - 2y e^{y^2}) dy = 0$

Solution

$$M = (x \cos xy - 2y e^{y^2}) dy \quad \therefore \frac{dM}{dx} = -xy \sin xy + \cos xy$$

$$N = (y \cos xy + e^x) dx \quad \therefore \frac{dN}{dy} = -xy \sin xy + \cos xy$$

$$\therefore \frac{\partial M}{\partial x} = \frac{\partial N}{\partial y} \quad \therefore \text{exact} \Rightarrow$$

$$\int (y \cos xy + e^x) dx + \int (x \cos xy - 2ye^{y^2}) dy = 0$$

$$\frac{y \sin xy}{y} + e^x + \frac{x \sin xy}{x} - 2 = 0$$

$$\sin xy + e^x + x \sin xy = 0$$

$$\begin{array}{r} y^2 \\ e^{y^2} \\ \downarrow \\ 2y \\ 1 \\ 0 \\ \hline 4y^2 \end{array}$$



ExactForm

$$M(x,y) dx + N(x,y) dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

∴ Exact type

طريق اكل ← نكمل طرفي المعادلة بحذف المتكرر.

Ex:-  $(x^3 + 3xy^2) + (3x^2y + y^3) \frac{dy}{dx} = 0$

$$M = x^3 + 3xy^2 \quad \therefore M_y = 0 + 6xy$$

$$N_x = 3x^2y + y^3 \quad \therefore N_x = 6xy + 0$$

$$\therefore M_y = N_x$$

$$\Rightarrow \int (x^3 + 3xy^2) dx + (3x^2y + y^3) dy = 0$$

$$\frac{x^4}{4} + \frac{3x^2y^2}{2} + \frac{3x^2y^2}{2} + \frac{y^4}{4} = C$$

نلاحظ ان المتكرر

$$\therefore \frac{x^4}{4} + 3 \frac{x^2y^2}{2} + \frac{y^4}{4} = C$$



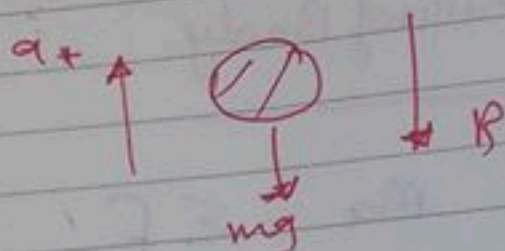




$$ma = -(mg + R)$$

$$a = -\left(g + \frac{R}{m}\right)$$

$$\rightarrow \text{at } T=0 \quad v=v_0$$



2 Cooling temperature :-

تغير الحرارة  
بالنسبة  
للمزج

$$\frac{dT}{dt} = -K(T - T_m)$$

↓  
الوسط 24 ك

قانون نيوتن للتبريد.

ex:- a metal bar at temperature of  $100^\circ\text{F}$   
is placed in a room at constant temp of  $0^\circ\text{F}$   
is after  $20 \text{ min}$   $T = 50^\circ\text{F}$  find

a-  $t$  for  $25^\circ\text{F}$  ??

b-  $T$  after  $10 \text{ min}$  ?

Solution

$$\frac{dT}{dt} = -K(T - 0) \quad \therefore \frac{dT}{dt} = -KT$$

$$\therefore \frac{1}{T} dT = -K dt$$

$$\therefore \ln T = -kt + C_1 = \ln A$$

$$\therefore T = C e^{-kT}$$

$$\rightarrow \text{at } T=0 \quad T=100^\circ \quad \therefore 100 = C e^{-0} \quad \therefore \boxed{C=100}$$

$$\therefore T = 100 e^{-kT}$$

$$\rightarrow \text{at } T=20 \text{ mi} \quad T=50^\circ$$

$$\therefore 50 = 100 e^{-k \times 20} \quad \therefore -20k = \ln\left(\frac{1}{2}\right)$$

$$\therefore k = 0.0346$$

$$\therefore T = 100 e^{-0.0346T}$$

$$\text{a) } \rightarrow 50 = 100 e^{-0.0346T} \quad \therefore T = \checkmark$$

$$\text{b) } \rightarrow T = 100 e^{-0.0346 \times 10} \quad \therefore T = \checkmark$$

### [3] orthogonal Trajectories :-

→ Given variables

→ Req orthogonal trajectories

→ Solution orthogonal trajectories

$$y'_2 = -\frac{1}{y'_1}$$



ex:- 1 find the o.t of the family of

Curves  $x^2 + y^2 = c^2$  ??

{Solution}

$$x^2 + y^2 = c^2$$

$$\therefore 2x + 2yy' = 0$$

$$\therefore y' = -\frac{x}{y}$$

$$\therefore y'_2 = -\frac{1}{y'}$$

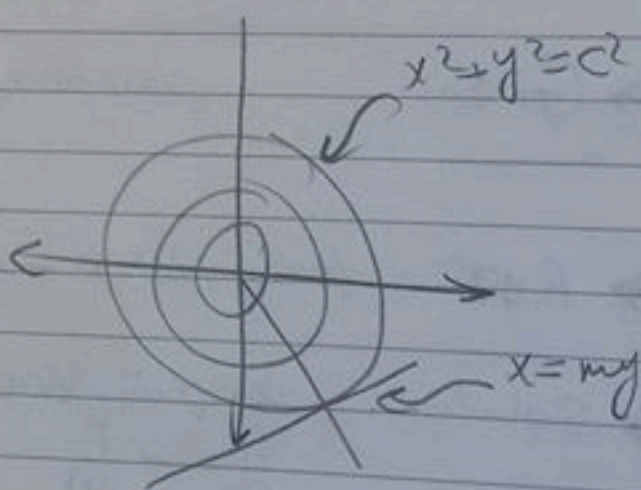
$$\frac{-1}{y'} = -\frac{x}{y}$$

$$\therefore \frac{dy}{dx} = \frac{x}{y}$$

$$\therefore \int \frac{1}{x} dx = \int \frac{1}{y} dy$$

$$\ln x = \ln y + \ln m$$

$$x = my$$



(2) Find the o.t of the family of Curve  $S$   
 $x^2 - y^2 = c^2$  (solution)

$$2x - 2yy' = 0$$

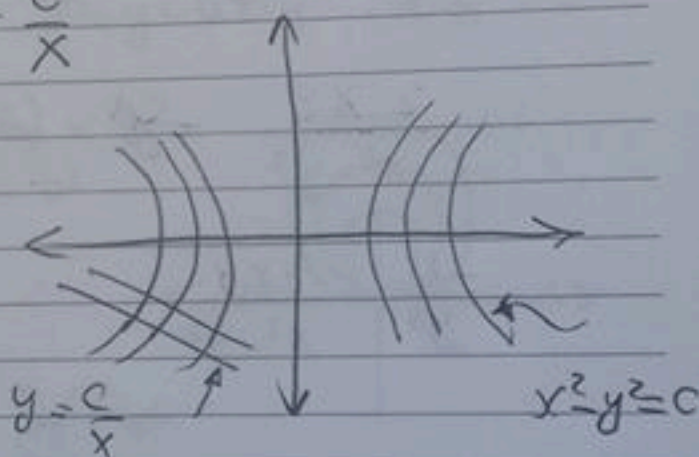
$$y' = \frac{2x}{2y} = \frac{x}{y} \quad \therefore \frac{1}{y'} = \frac{y}{x}$$

$$\therefore \frac{dy}{dx} = \frac{x}{y}$$

$$\ln y = -\ln x + \ln c$$

$$\ln y + \ln x = \ln c$$

$$y = \frac{c}{x}$$





(2) Find the O.T. of the family of curves  
 $x^2 + y^2 = 2by \Rightarrow b = \frac{x^2 + y^2}{2y}$

Solution

$$2x + 2yy' = 2by' \rightarrow x + yy' = \left(\frac{x^2 + y^2}{2y}\right)y'$$

$$\therefore x = y' \left( \frac{x^2 + y^2}{2y} - y \right)$$

$$\text{Put } y' = \frac{-1}{y}$$

$$x = \frac{-1}{y} \left( \frac{x^2 + y^2}{2y} - y \right) = \left( \frac{x^2 + y^2 - 2y^2}{2y} \right) y'$$

$$\therefore \frac{2xy}{x^2 - y^2} = y' \rightarrow \text{homogeneous function}$$

$$\therefore \frac{-1}{y} = \frac{2xy}{x^2 - y^2} \quad \therefore \frac{y^2 - x^2}{2xy} = \frac{dy}{dx}$$

$$\Rightarrow \text{Put } y = vx$$

$$y' = \frac{dv}{dx} x + v$$

$$\therefore \frac{v^2 x^2 - x^2}{2vx^2} = \frac{dv}{dx} x + v$$

$$\therefore \frac{v^2 - 1}{2v} - v = x \frac{dv}{dx} \quad \therefore \frac{v^2 - 1 - 2v^2}{2v} = x \frac{dv}{dx}$$

$$\therefore \frac{-(1 - v^2)}{2v} = x \frac{dv}{dx}$$

$$= \int \frac{-1}{x} dx = \int \frac{2v}{1+v^2} dv$$

$$-\ln x + \ln C = \ln(1+v^2)$$

$$\therefore C = X(1+v^2)$$

$$C = X(1 + (\frac{y}{x})^2)$$

$$C = \left(\frac{y^2 + x^2}{x^2}\right) x$$

$$Cx = y^2 + x^2$$

مركزه  
في  
15  
مركزه  
في  
2/1

$$x^2 + y^2 = 2by \quad \Rightarrow \quad x^2 + y^2 - by + b^2 - b^2 = 0$$

$$x^2 + (y^2 - by + b^2) = b^2$$

$$x^2 + (y-b)^2 = b^2$$

$\therefore$

$$\Rightarrow x^2 + y^2 = Cx$$

$$x^2 + y^2 - Cx + \frac{C^2}{4} - \frac{C^2}{4} = 0$$

$$\therefore y^2 + \left(x - \frac{C}{2}\right)^2 = \left(\frac{C}{2}\right)^2$$

